Coefficient of Variation:

This relative measure is known as coefficient of variation (C.V). The credit for coefficient of variation goes to Karl Pearson. It is not expressed in absolute values but expressed in percentage. For calculating C.V, none of the variables should be -ve, otherwise it cannot be calculated.

$$C.V = \frac{\sigma}{\bar{x}} \times 100$$

The per kilogram retail prices of apples in Delhi in different months were found as under:

Month:	Sep	Oct	Nov	Dec	Jan	Feb	Mar
Prices:	60	50	35	45	48	55	64

Calculate Variance, S.D & C.V.

Solution:

Month	Price (x)	$(x-\bar{x})^2$
Sep	60	81
Oct	50	1
Nov	35	256
Dec	45	36
Jan	48	9
Feb	55	16
Mar	64	169
	$\Sigma x = 357$	568

$$\bar{x} = \frac{\Sigma x}{n} = \frac{357}{7} = Rs\,51/-$$

$$\sigma = \sqrt{\frac{\Sigma(x-\bar{x})^2}{n}} = \sqrt{\frac{568}{7}} = \sqrt{81.14} = 9 \text{ (app.)}$$

$$\sigma^2 = 81.14.$$

$$C.V = \frac{\sigma}{\bar{x}} \times 100 = \frac{9}{51} \times 100 = 17.65\%.$$

The above procedure is not very suitable when mean is in fractional value. In such a case, first find the sum of all observations, square all observations & total the squared observation. Apply the following formula to obtain S.D.

$$\sigma = \sqrt{\frac{\Sigma x^2}{n} - \left(\frac{\Sigma x}{n}\right)^2} \quad \text{or} \quad \sqrt{\frac{\Sigma x^2}{n} - (\bar{x})^2}.$$

Short cut method:

S.D is also calculated by a shortcut method. From an assumed mean, deviation of all values is taken (similar to the procedure used in computing mean), total these deviations, square all deviations, total the squared deviation & apply the formula.

$$\sigma = \sqrt{\frac{\Sigma d^2}{n} - \left(\frac{\Sigma d}{n}\right)^2} \quad \text{where } d = x - A.$$

A being assumed mean.

Incase of discrete series, the basic procedure remains the same as explained above but every term is multiplied by the corresponding frequency. The modified formulas are shown below.

$$\sigma = \sqrt{\frac{\Sigma f(x-\bar{x})^2}{n}}$$

$$\sigma = \sqrt{\frac{\Sigma fx^2}{n} - \left(\frac{\Sigma fx}{n}\right)^2} \quad \text{or} \quad \sqrt{\frac{\Sigma fx^2}{n} - (\bar{x})^2}$$

$$\sigma = \sqrt{\frac{\Sigma fd^2}{n} - \left(\frac{\Sigma fd}{n}\right)^2}$$

A travel & tourist consultancy firm offers a variety of four packages for different destinations in India both to the foreign tourists as well as domestic tourists. In the tourist season of the last winter the following bookings were made by the firm:

Tariff (Rs. thousands):	4	7	10	15	25	50
No. of bookings:	80	75	40	32	15	8

Calculate the following:

i) Total amount of booking

ii) Mean amount per booking.

iii) S.D per booking

iv) Coefficient of variation

Solution:

x (Amt in 1000)	f	fx	$f(x-\bar{x})^2$
4	80	320	2,880
7	75	525	675
10	40	400	0
15	32	480	800
25	15	375	3375
50	8	400	12,800
	$n=250$	2,500	20,530

$n = 250$, $\Sigma fx = 2,500$, $\Sigma f(x-\bar{x})^2 = 20,530$

i) Total amount of booking = Rs 2,500 thousands
(or)
Rs 25,00,000.

ii) Mean amount per booking $\bar{x} = \frac{\Sigma fx}{n} = \frac{2500}{250} = 10$
= Rs 10,000.

iii) $S.D = \sqrt{\frac{\Sigma f(x-\bar{x})^2}{n}} = \sqrt{\frac{20,530}{250}} = 9.062$ = Rs 9062.

iv) $C.V = \frac{\sigma}{\bar{x}} \times 100$

$= \frac{9.062}{10} \times 100 = 90.62\%$.

Number of Patients increased tremendously after every rainy season. They have to spend lot of money to buy medicines. This increases the sales of pharmaceutical companies. The amount of bills of medicines dispatched to retail chemists in the preceding July by a pharmaceutical company is given below:

Amount (Rs 100):	4	8	10	13	18	24	30
No. of Bills:	18	22	45	60	52	36	17

Calculate S.D & C.V for the above data by direct method.

Solution:

x	f	fx	fx^2
4	18	72	288
8	22	176	1408
10	45	450	4500
13	60	780	10,140
18	52	936	16,848
24	36	864	20,736
30	17	510	15,300
	250	3,788	69,220.

$n = 250$, $\Sigma fx = 3,788$ $\Sigma fx^2 = 69,220.$

$$\sigma = \sqrt{\frac{\Sigma fx^2}{n} - \left(\frac{\Sigma fx}{n}\right)^2} = \sqrt{\frac{69,220}{250} - \frac{3788}{250}}$$

$$= \sqrt{276.88 - 229.58}$$

$$= \sqrt{47.3} = \text{RS } 6.88 \text{ thousand.}$$

$$\bar{x} = \frac{\Sigma fx}{n} = \frac{3,788}{250} = \text{Rs } 15.15 \text{ thousand}$$

$$C.V = \frac{\sigma}{\bar{x}} \times 100 = \frac{6.88}{15.15} \times 100 = 45.41\%.$$

Sizes of land holdings of small farmers in a district are given below. From these data calculate i) Mean ii) S.D iii) C.V. Apply Shortcut method.

Farm size (Acres):	5	8	10	12	15	25	50	75
No. of Farms:	24	35	42	58	63	16	9	3.

Solution:

x	f	$d = (x - 25)$	fd	fd^2
5	24	-20	-480	9,600
8	35	-17	-595	10,115
10	42	-15	-630	9,450
12	58	-13	-754	9,802
15	63	-10	-630	6,300.
25	16	0	0	0
50	9	25	225	5,625
75	3	50	150	7,500
	250		-2,714	58,392.

$n = 250$, $\Sigma fd = -2,714$ $\Sigma fd^2 = 58,392$.

$$\bar{x} = A + \frac{\Sigma fd}{n} = 25 - \frac{2,714}{250} = 25 - 10.856 = 14.144 \text{ acres}$$

$$\sigma = \sqrt{\frac{\Sigma fd^2}{n} - \left(\frac{\Sigma fd}{n}\right)^2} = \sqrt{\frac{58,392}{250} - \left(\frac{-2,714}{250}\right)^2}$$

$$= \sqrt{233.57 - 117.85} = \sqrt{115.72}$$

$$= 10.76 \text{ acres}.$$

$$C.V = \frac{\sigma}{\bar{x}} \times 100 = \frac{10.76}{14.144} \times 100 = 76.07\%.$$

S.D is calculated by the following formula in case of continuous series:

$$\sigma = \sqrt{\frac{\Sigma fd'^2}{n} - \left(\frac{\Sigma fd'}{n}\right)^2} \times h.$$

where $d' = \frac{x - A}{h}$ A = assumed mean.

h = class interval.

Steps required to calculate S.D in case of continuous series:

i) Find mid-point for each class

ii) Total the frequency column & call the total n.

iii) Take step deviation 'd' as explained above

iv) Multiply 'd' by each corresponding frequency & total the column, call it $\Sigma fd'$

v) Multiply fd' by d' to obtain fd'^2 & then total the column call it $\Sigma fd'^2$

vi) Put these values in the above stated formula & then multiply the square root value by the class interval h.

The HRD section of a company conducted an examination of those candidates who applied for the post of computer operator. In all 150 candidates appeared in the written examination. Their scores, out of 100, are given in the following table. Calculate S.D & Coefficient of variation of the scores.

Scores	No. of Candidates
0-10	8
10-20	15
20-30	17
30-40	28
40-50	25
50-60	24
60-70	18
70-80	9
80-90	6

Solution:

Score	f	x (Mid Point)	d'	fd'	fd'^2
0-10	8	5	-4	-32	128
10-20	15	15	-3	-45	135
20-30	17	25	-2	-34	68
30-40	28	35	-1	-28	28
40-50	25	45	0	0	0
50-60	24	55	1	24	24
60-70	18	65	2	36	72
70-80	9	75	3	24	81
80-90	6	85	4	24	96

$n = \Sigma f = 150 \qquad \Sigma fd' = -28 \qquad \Sigma fd'^2 = 632$

$$\bar{x} = A + \frac{\Sigma fd'}{n} \times h = 45 - \frac{28}{150} \times 10 = 43.13$$

$$\sigma = \sqrt{\frac{\Sigma fd'^2}{n} - \left(\frac{\Sigma fd'}{n}\right)^2} \times h = \sqrt{\frac{632}{150} - \left(\frac{-28}{150}\right)^2} \times 10$$

$$= \sqrt{4.213 - 0.035} \times 10$$

$$= 20.44$$

$$C.V = \frac{\sigma}{\bar{x}} \times 100 = \frac{20.44}{43.13} \times 100 = 47.39\%.$$

Calculate variance & coefficient of variation for the following data:

weights	No. of students
60-64	10
65-74	40
75-79	35
80-84	22
85-94	26
95-99	10
100-104	7

Solution: Before further calculation, the series should be converted into an exclusive series.

Weights	No. of students (f)	Mid-Point x	d = x - A	fd	fd^2
59.5 - 64.5	10	62	-20	-200	4000
64.5 - 74.5	40	69.5	-12.5	-500	6250
74.5 - 79.5	35	77	-5	-175	875
79.5 - 84.5	22	82	0	0	0
84.5 - 94.5	26	89.5	7.5	195	1462.5
94.5 - 99.5	10	97	15	150	2250
99.5 - 104.5	7	102	20	140	2800
	150			-390	17,637.5

$n = 150$, $\Sigma fd = -390$, $\Sigma fd^2 = 17,637.5$

$$\bar{x} = A + \frac{\Sigma fd}{n} = 82 - \frac{390}{50} = 79.4$$

$$\sigma^2 = \frac{\Sigma fd^2}{n} - \left(\frac{\Sigma fd}{n}\right)^2 = \frac{17637.5}{150} - \left(-\frac{390}{150}\right)^2$$

$$= 117.58 - 6.76 = 110.82.$$

$$\sigma = \sqrt{110.82} = 10.53$$

$$C.V = \frac{\sigma}{\bar{x}} \times 100 = \frac{10.53}{79.4} \times 100 = 13.26\%.$$